Exercise 1.47

Find the angle between each of the following pairs of vectors:

(a) $\vec{\boldsymbol{A}} = -2.00\hat{\boldsymbol{i}} + 6.00\hat{\boldsymbol{j}}$ and $\vec{\boldsymbol{B}} = 2.00\hat{\boldsymbol{i}} - 3.00\hat{\boldsymbol{j}}$

(b) $\vec{\boldsymbol{A}} = 3.00\hat{\boldsymbol{i}} + 5.00\hat{\boldsymbol{j}}$ and $\vec{\boldsymbol{B}} = 10.00\hat{\boldsymbol{i}} + 6.00\hat{\boldsymbol{j}}$

(c) $\vec{\boldsymbol{A}} = -4.00\hat{\boldsymbol{i}} + 2.00\hat{\boldsymbol{j}}$ and $\vec{\boldsymbol{B}} = 7.00\hat{\boldsymbol{i}} + 14.00\hat{\boldsymbol{j}}$

Solution

The dot product of two vectors, $\mathbf{A}$ and $\mathbf{B}$, can be written in terms of the angle θ between them.

$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}||\mathbf{B}| \cos\theta$$

Solve for θ.

$$\cos\theta = \frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}||\mathbf{B}|}$$

$$\begin{aligned}\theta &= \cos^{-1}\left(\frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}||\mathbf{B}|}\right) \\ &= \cos^{-1}\left(\frac{A_x B_x + A_y B_y}{\sqrt{A_x^2 + A_y^2}\sqrt{B_x^2 + B_y^2}}\right)\end{aligned}$$

If $\mathbf{A} = -2.00\hat{\boldsymbol{i}} + 6.00\hat{\boldsymbol{j}}$ and $\mathbf{B} = 2.00\hat{\boldsymbol{i}} - 3.00\hat{\boldsymbol{j}}$, then

$$\theta = \cos^{-1}\left[\frac{(-2.00)(2.00) + (6.00)(-3.00)}{\sqrt{(-2.00)^2 + (6.00)^2}\sqrt{(2.00)^2 + (-3.00)^2}}\right] \approx 165^\circ.$$

If $\mathbf{A} = 3.00\hat{\boldsymbol{i}} + 5.00\hat{\boldsymbol{j}}$ and $\mathbf{B} = 10.00\hat{\boldsymbol{i}} + 6.00\hat{\boldsymbol{j}}$, then

$$\theta = \cos^{-1}\left[\frac{(3.00)(10.00) + (5.00)(6.00)}{\sqrt{(3.00)^2 + (5.00)^2}\sqrt{(10.00)^2 + (6.00)^2}}\right] \approx 28.1^\circ.$$

If $\mathbf{A} = -4.00\hat{\boldsymbol{i}} + 2.00\hat{\boldsymbol{j}}$ and $\mathbf{B} = 7.00\hat{\boldsymbol{i}} + 14.00\hat{\boldsymbol{j}}$, then

$$\theta = \cos^{-1}\left[\frac{(-4.00)(7.00) + (2.00)(14.00)}{\sqrt{(-4.00)^2 + (2.00)^2}\sqrt{(7.00)^2 + (14.00)^2}}\right] = \cos^{-1}(0) = 90.0^\circ.$$